Electoral Competition under Costly Policy Implementation*

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Abstract

This paper analyzes unidimensional electoral competition between two policy-motivated candidates, assuming that the elected candidate has to incur an idiosyncratic policy implementation cost. Think of this as a chairman election in an academic department, a parent-teacher association, or a condominium association board, where office holding is perceived as a cost –not as a benefit– by the election's winner. We prove that in such a game, a pure strategy Nash equilibrium is guaranteed to exist even when the policy implementation costs are heterogeneous, as long as they are not very large. We also provide characterization, comparative statics and welfare analysis of the equilibrium in the case of Euclidean preferences. In particular, we show that equilibrium strategies are divergent and in general not symmetric around the expected bliss point of the median voter. A higher policy implementation cost makes the candidate propose a more extreme policy and lowers his electoral chances. Naturally, the voter's welfare decreases when the candidates' implementation costs increase.

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1. Introduction

This paper analyzes a spatial model of electoral competition between two policy-motivated candidates in which an elected candidate has to incur a cost of policy implementation. Think of this as an office which brings no benefits but instead implies costs that the holder has to bear in order to gain influence on policy outcomes. One possible example is a chairman position in an academic department. It is usually associated with high opportunity costs and the lack of additional financial compensation. And there is often disagreement among candidates about what departmental policies to implement. Another example is a chairman position in a parent-teacher association (PTA). It requires a lot of effort, time and commitment. And there are many different views of how it should be run. The exact same points made about a PTA also apply in the case of a head of a condominium association board (or of similar volunteer organizations). In all these examples, candidates care both about policy and office but the value of office is negative (instead of the more traditional assumption of it being positive).

The question of equilibrium existence in electoral competition models with two office- and policy-motivated candidates is still open in the literature. We are not aware of equilibrium existence results for the case of negative value of office (as in our setting). To the best of our knowledge, Saporiti (2008) provides the most general existence result for the case of positive value of holding office. He assumes Euclidean preferences and proves that there exists a pure strategy Nash equilibrium only if candidates have the same valuations for office. Otherwise, the existence of a pure strategy Nash equilibrium is not guaranteed. Instead, in the case of negative value of office, we establish existence of a pure strategy Nash equilibrium much more generally. We assume concave preferences and show that there exists a pure strategy Nash equilibrium even when the costs of policy implementation are heterogeneous as long as they are not very large.

We describe now our setting and provide intuitions for our existence result. We study a spatial model of electoral competition with one-dimensional policy space (which is taken to be the real line), electoral uncertainty and full commitment. There are two candidates who run in an election. They care about policy and also about costs an elected candidate has to incur in order to implement his announced policy. We assume that the candidates have heterogeneous concave preferences over the policy and face heterogenous fixed costs of policy implementation. They don't observe the median voter's preferred policy but share a common prior about it, in particular, a log-concave symmetric distribution with zero mean.

The candidates simultaneously announce policies they want to implement if elected. We assume full commitment such that an elected candidate commits to follow the policy he announced beforehand. Facing their announcements, a median voter appoints one of them for policy implementation. The elected candidate implements his announcement and incurs the cost of policy implementation.

Our first result is existence of a pure strategy Nash equilibrium in the case of sufficiently small costs of policy implementation. In this equilibrium, the candidates announce divergent policies which are more moderate than their corresponding ideal points. The intuition for this existence result is as follows. Fix a strategy of one candidate, say candidate 1, close to his ideal point and consider the expected payoff of candidate 2. The closer candidate 2's announcement to candidate 1's, the lower his expected payoff is. Indeed, his election probability increases (which is payoff reducing because of the negative value of office) and his expected payoff from policy decreases (because he commits to a policy which is more far away from his bliss point). This implies the quasi-concavity of candidate 2's payoff function as well as the existence and uniqueness of his best response to candidate 1's strategy. Our existence result follows then. Notice that this intuition does not work in the case of positive value of holding office analyzed by Saporiti (2008). The reason is that an increase in candidate 2's election probability is actually payoff increasing because of the positive value associated with office. Therefore, the expected payoff of candidate 2 is not always decreasing when his announcement approaches candidate 1's strategy. It follows that his payoff function is generally not quasi-concave in this case.

Our existence result is established for the case of sufficiently small costs of policy implementation, which guarantees that the equilibrium announcements are more moderate than the candidates' ideal policies. In the case of high costs, policy implementation becomes too expensive for the candidates so that they prefer to lose the election. Then given one candidate's strategy, the other candidate has an incentive to announce a more extreme policy in order to increase his chances of losing the election and so saving the high cost of office. It implies that the existence of pure strategy Nash equilibrium is not guaranteed in the case of high costs.

Given our existence result, we turn then to equilibrium characterization and comparative statics analysis for the case of Euclidean preferences. We implicitly characterize equilibrium announcements which are in general not symmetric around the expected ideal point of the median voter. Only in the case of homogeneous costs, they are symmetric. We show that the higher the candidate's cost of policy implementation, the closer his announcement is to his ideal point. In other words, the more extreme policy he proposes. However, the effect on his competitor's policy depends on the costs' magnitude. An increase in a lower cost makes the competitor's policy more moderate while an increase in a higher cost makes it more extreme. As for the candidate's electoral chances, they decrease as his cost of policy implementation raises. This in turn increases his competitor's chances.

We study next the welfare properties of this equilibrium. The effect of costs on the median voter's welfare depends on the relationship between the electoral uncertainty and costs of policy implementation. In the case of equal costs, we show that the voter's welfare decreases with the cost of policy implementation. Intuitively, higher costs makes the candidates announce more extreme policies which is welfare reducing for the voter.

When the distribution of the median voter's bliss point converges to the certainty case, the equilibrium announcements are symmetric around her expected bliss point (both for homogeneous and heterogeneous costs of policy implementation). We have also analyzed the case without electoral uncertainty in which the median voter is modeled as a strategic player. In this case, there is a continuum of subgame perfect equilibria in which the candidates propose symmetric policies and the median voter appoints the lower cost candidate. These equilibria are not payoff equivalent. Interestingly, the equilibrium of the electoral uncertainty case converges to one equilibrium from this continuum, in particular, to the one in which the lower cost candidate gets the highest payoff while the higher cost candidate gets the lowest payoff.

The remainder of the paper is organized as follows. The next Section describes the related literature. Section 3 describes the model. Section 4 proceeds with equilibrium existence. Section 5 presents the results for equilibrium characterization, comparative statics and welfare analysis in the case of Euclidean preferences. Finally, Section 6 concludes the paper.

2. Related Literature

Our paper contributes to the literature on spatial competition between two players with full commitment, which originated in the location model of Hotelling (1929). Electoral competition interpretation goes back to the seminal work of Downs (1957), who assumed purely office-motivated candidates. Under the assumption of single-peaked voters' preferences, Downs (1957) established existence and uniqueness of a pure strategy Nash equilibrium with convergence. In this equilibrium, both candidates announce the median voter's preferred policy (Hinich 1977). This convergence result prevails in the case of electoral uncertainty in which the candidates announce the expected bliss point of the median voter (Calvert 1985).

A further step was taken by Wittman (1977, 1983, 1990), Calvert (1985) and Roemer (1994), who considered purely policy-motivated candidates. It has been shown that under full commitment, two policy-motivated candidates announce convergent platforms if the distribution of the voters' ideal policies is known (Wittman 1977, Calvert 1985, Roemer 1994, Duggan and Fey 2005, Bernhardt et al. 2009). The case of electoral uncertainty has been studied by Calvert (1985) and later by Roemer (1997, 2001), who established existence of a pure strategy Nash equilibrium with divergent policy announcements.

Our model is mostly related to the so-called *hybrid* case in which the candidates care both about the policy and office. The literature has addressed the hybrid case under the assumption of positive value of holding office. To the best of our knowledge, Calvert (1985) was the first to show existence and uniqueness of a pure strategy Nash equilibrium with convergence under assumption of electoral certainty. The hybrid case with electoral uncertainty has been firstly addressed by Ball (1999). He mentioned that the game might not have a Nash equilibrium in pure strategies but didn't provide equilibrium characterization (either in pure or mixed strategies). As we explained in Introduction, this game has been later reconsidered by Saporiti (2008), who provided equilibrium existence and characterization for the hybrid case with positive value of office and electoral uncertainty.

To the best of our knowledge, the literature has not addressed the hybrid case with negative value of holding office which we study here. Most literature on spatial competition has disregarded costs that an elected candidate has to incur in office in order to implement his announced policy.[1] We are aware just of few papers which somewhat introduced such a cost. Bilodeau and Slivinski (1996) and Messner and Polborn (2004) considered vertically differentiated agents which decide whether to provide a costly public good. In contrast to the present paper, agents in these studies are endowed with a variety that they would provide if appointed and so cannot choose other varieties. In turn, Hirsch and Shotts (2015) analyzed a contest game in which two candidates submit proposals that have both an ideological component and a costly quality component. In their model, however, all contest participants incur costs of crafting proposals while in the present study the cost is carried by an appointed candidate alone.

3. Model

Consider a spatial model of electoral competition between two policy-motivated candidates under costly policy implementation. The set of feasible policy outcomes is $\mathbb{R}$. There are

[1] This policy implementation cost is somewhat similar to a private cost of providing a public good in the literature on voluntary provision of public goods (Bergstrom et al. 1986, Bliss and Nalebuff 1984, Andreoni 1988, Cornes and Sandler 1996). The novelty of this paper is that the public good (which is policy in our setting) is assumed to be differentiated.

two candidates, 1 and 2, who run in an election. Their costs of policy implementation are common knowledge and denoted by $C_1 > 0$ and $C_2 > 0$, respectively.

The candidates simultaneously announce policies x_1 and x_2 that they want to implement if elected. Then, facing their announcements x_1 and x_2, a median voter elects one of them for policy implementation. If indifferent, she appoints each of them with equal probability. An elected candidate implements his announcement and incurs the cost of policy implementation. Note that we assume full commitment such that once elected, a candidate implements a policy he has announced beforehand.

The median voter and the candidates differ in their policy preferences. The median voter's most preferred policy is ρ. She knows her preferred policy ρ while the candidates don't observe ρ. They share a common prior $F(\cdot)$ about ρ, where $F(\cdot)$ is a log-concave symmetric distribution with zero mean and full support.[2,3] The candidates' most preferred policies are α_1 and α_2 which are common knowledge and such that $\alpha_1 < 0 < \alpha_2$.

The median voter and the candidates have utility loss functions which decrease with the distance between the implemented policy and one's ideal point. Formally, the utility of an individual with ideal point $\chi \in \mathbb{R}$ when policy $x \in \mathbb{R}$ is implemented is given by $v(|x - \chi|)$, where $v : [0, +\infty) \to \mathbb{R}$ is a strictly decreasing, twice-differentiable and concave function. Thus, they want the implemented policy to be close to their bliss point.

Median Voter's Problem The median voter's problem is to elect a candidate for policy implementation given the announcements x_1 and x_2. Intuitively, the median voter elects a candidate whose announced policy generates a higher level of utility for her. Formally, the median voter elects candidate 1 if

$$v\left(|x_1 - \rho|\right) > v\left(|x_2 - \rho|\right),$$

which amounts to $\rho < \frac{x_1+x_2}{2}$ for $x_1 < x_2$, and to $\rho > \frac{x_1+x_2}{2}$ for $x_1 > x_2$. The median voter elects candidate 2 if

$$v\left(|x_1 - \rho|\right) < v\left(|x_2 - \rho|\right),$$

which amounts to $\rho > \frac{x_1+x_2}{2}$ for $x_1 < x_2$, and to $\rho < \frac{x_1+x_2}{2}$ for $x_1 > x_2$. Finally, the median voter elects each of the candidates with equal probability if

$$v\left(|x_1 - \rho|\right) = v\left(|x_2 - \rho|\right),$$

[2] F is log-concave if $\frac{\partial^2 \ln F(x)}{\partial x^2} < 0$ and $\frac{\partial^2 \ln[1-F(x)]}{\partial x^2} < 0$ for every $x \in \mathbb{R}$. Hence, our definition of log-concavity implies that F is twice-differentiable in $\mathbb{R}$ as well.

[3] Modelling electoral uncertainty with a prior distribution $F(\cdot)$ is common in the political competition literature (see Martimort and Semenov 2008, Saporiti 2008).

which holds for $x_1 = x_2$, or $\rho = \frac{x_1+x_2}{2}$ for $x_1 \neq x_2$. The candidates don't observe ρ but share a common prior $F(\cdot)$ about ρ. Therefore, they assign probability

$$p_1(x_1, x_2) = \begin{cases} \Pr\left(\rho < \frac{x_1+x_2}{2}\right) = F\left(\frac{x_1+x_2}{2}\right) & \text{if} \quad x_1 < x_2, \\ \Pr\left(\rho > \frac{x_1+x_2}{2}\right) = 1 - F\left(\frac{x_1+x_2}{2}\right) & \text{if} \quad x_1 > x_2, \\ \frac{1}{2} & \text{if} \quad x_1 = x_2, \end{cases}$$

to the median voter electing candidate 1, and probability

$$p_2(x_1, x_2) = 1 - p_1(x_1, x_2)$$

to the median voter electing candidate 2 in the election.

Given these probabilities, the candidates' utility functions $\Pi_i(x_1, x_2)$, $i = 1, 2$, are given by

$$\begin{aligned} \Pi_1(x_1, x_2) &= p_1(x_1, x_2)\left(v(|x_1 - \alpha_1|) - C_1\right) + p_2(x_1, x_2)\, v(|x_2 - \alpha_1|), \\ \Pi_2(x_1, x_2) &= p_1(x_1, x_2)\, v(|x_1 - \alpha_2|)) + p_2(x_1, x_2)\left(v(|x_2 - \alpha_2|) - C_2\right). \end{aligned}$$

The candidates simultaneously announce policies x_1 and x_2 to maximize their corresponding utilities. We search for pure strategy Nash equilibria of this game. We call an equilibrium, (x_1^*, x_2^*), interior when $\alpha_1 < x_1^* < x_2^* < \alpha_2$. This use of the term "interior" is not common: an equilibrium is usually called interior when players use strategies from the interior of their strategy sets. In this paper, the strategy space of each player is $\mathbb{R}$, and hence, we call an equilibrium interior when both candidates use strategies in the interior of the interval defined by their ideal points.

4. Equilibrium Existence

In this section, we establish equilibrium existence. The following proposition presents the existence result.

Proposition 1. *For every α_1, α_2 and $F(\cdot)$, a pure strategy Nash equilibrium, (x_1^*, x_2^*), exists when the costs of policy implementation are sufficiently small.*

Proof To prove existence of an equilibrium we need to consider a restricted game. We consider a positive number $d = \max\{-k_1, k_2\}$ and we assume that candidate 1 is allowed to announce a policy in $[\alpha_1, -d]$ and that candidate 2 is allowed to announce a policy in $[d, \alpha_2]$. The value k_i is given by $v(|k_i - \alpha_i|) - C_i = v(|-k_i - \alpha_i|)$. It pins down the location beyond which candidate i would unambiguously decrease his payoff when approaching further his –symmetrically located– competitor. Our assumptions regarding v guarantee that, for sufficiently small costs of policy implementation: a) a unique k_i exists for each i; b) it is more central than i's ideal policy; and c) it monotonically converges to zero when C_i goes to zero. More broadly, when $x_{-i} \times \alpha_{-i} > 0$ and implementation costs are sufficiently small, one can further define $x_i(x_{-i}, C_i)$ as the location that makes a candidate completely indifferent between winning and losing, and is obtained from solving $v(|x_i(x_{-i}, C_i) - \alpha_i|) - C_i = v(|x_{-i} - \alpha_i|)$.

Our aim is to show first that the restricted game admits a pure strategy equilibrium and then argue that this equilibrium should extend to the unrestricted version of the game; that is, when both candidates are free to propose any policy in $\mathbb{R}$. Obviously, such a version of the game may exist only when the implementation costs are sufficiently small. Formally, only when $d < \min\{-\alpha_1, \alpha_2\}$.

Step 1. In the restricted game, we have that

$$\frac{\partial \Pi_1(x_1,x_2)}{\partial x_1} = v'(x_1 - \alpha_1) F\left(\frac{x_1+x_2}{2}\right) + \frac{1}{2}(-C_1 + v(x_1 - \alpha_1) - v(x_2 - \alpha_1)) F'\left(\frac{x_1+x_2}{2}\right).$$

We observe that $\frac{\partial \Pi_1(x_1,x_2)}{\partial x_1} |_{x_1=\hat{x}_1} = 0$ if and only if

$$\frac{v'(x_1-\alpha_1)}{\frac{1}{2}(C_1 - v(x_1-\alpha_1) + v(x_2-\alpha_1))} = \frac{F'\left(\frac{x_1+x_2}{2}\right)}{F\left(\frac{x_1+x_2}{2}\right)}$$

for some $x_1 = \hat{x}_1$. Notice that the first term is strictly increasing in x_1 (due to the fact that v is strictly decreasing and concave)[4] and that $\frac{F'\left(\frac{x_1+x_2}{2}\right)}{F\left(\frac{x_1+x_2}{2}\right)}$ is strictly decreasing in x_1 (due to log-concavity of F). Moreover, when $x_2 \in [d, \alpha_2]$,

$$\lim_{x_1 \to x_1(x_2,C_1)^-} \frac{\partial \Pi_1(x_1,x_2)}{\partial x_1} = v'(x_1 - \alpha_1) F\left(\frac{x_1+x_2}{2}\right) < 0$$

(obviously x_1 cannot approach $x_1(x_2, C_1)$ in the restricted game, but this observation is crucial for our argument). Therefore, $\Pi_1(x_1, x_2)$ is strictly quasi-concave in x_1 in the restricted game for any admissible x_2. That is, by Debreu (1952) it follows that this restricted game admits a Nash equilibrium in pure strategies, (x_1^*, x_2^*).

Now, we will argue that when $d \to 0$, none of the equilibrium strategies of the restricted game, x_1^* and x_2^*, converges to zero.

[4] To see this notice that its derivative with respect to x_1 is equal to $\frac{2[v'(x_1-\alpha_1)^2 - (C_1 - v(x_1-\alpha_1) + v(x_2-\alpha_1))v''(x_1-\alpha_1)]}{(C_1 - v(x_1-\alpha_1) + v(x_2-\alpha_1))^2}$, and since $v'(x_1 - \alpha_1) \neq 0$ and $v''(x_1 - \alpha_1) \leq 0$, it must be positive.

Step 2. Assume that $d \to 0$ and that the candidates play the Nash equilibrium of the restricted game, (x_1^*, x_2^*). If $(x_1^*, x_2^*) \to (0,0)$, then candidate 1 can deviate to $x_1 = \alpha_1$ and get a strictly larger payoff. Similarly, if $x_1^* \to 0$ and $x_2^* \to b > 0$ then candidate 1 can deviate to some non-degenerate[5] $k < 0$ and get a strictly larger payoff.[6] Therefore, there exists $\bar{d} > 0$ such that whenever $d < \bar{d}$, the Nash equilibrium of the restricted game, (x_1^*, x_2^*) is such that $(x_1^*, x_2^*) \in [\alpha_1, -d) \times (d, \alpha_2]$.

Next we argue that if (x_1^*, x_2^*) is an equilibrium of the restricted game, then a best response of candidate 1 in the unrestricted game to candidate 2 playing x_2^* cannot be strictly to the left of his ideal policy α_1. Similarly, the set of best responses of candidate 2 in the unrestricted game when candidate 1 plays x_1^* does not contain varieties strictly to the right of α_2.

Step 3. Assume that $x_1 < \alpha_1$. Then

$$\frac{\partial \Pi_1\left(x_1, x_2^*\right)}{\partial x_1} = -v'(-x_1 + \alpha_1) F\left(\frac{x_1 + x_2^*}{2}\right) + \frac{1}{2}(-C_1 + v(-x_1 + \alpha_1) - v(x_2^* - \alpha_1)) F'\left(\frac{x_1 + x_2^*}{2}\right).$$

When $d < \min\{-\alpha_1, \alpha_2\}$, we have that: a) $\lim_{x_1 \to \alpha_1^-} \frac{\partial \Pi_1\left(x_1, x_2^*\right)}{\partial x_1} > 0$ and b) $\frac{\partial \Pi_1\left(x_1, x_2^*\right)}{\partial x_1} |_{x_1 = \hat{x}_1} = 0$ if and only if

$$\frac{v'(-x_1 + \alpha_1)}{\frac{1}{2}(-C_1 + v(-x_1 + \alpha_1) - v(x_2^* - \alpha_1))} = \frac{F'\left(\frac{x_1 + x_2^*}{2}\right)}{F\left(\frac{x_1 + x_2^*}{2}\right)}$$

for some $x_1 = \hat{x}_1$. Notice that, the first term is strictly increasing in x_1 (with a similar reasoning to that of step 1) and $\frac{F'\left(\frac{x_1 + x_2^*}{2}\right)}{F\left(\frac{x_1 + x_2^*}{2}\right)}$ is strictly decreasing in x_1 (due to log-concavity of F). Hence, by $\lim_{x_1 \to \alpha_1^-} \frac{\partial \Pi_1\left(x_1, x_2^*\right)}{\partial x_1} > 0$ it follows that there exists at most one $\hat{x}_1 < \alpha_1$ for which $\frac{\partial \Pi_1(x_1, x_2)}{\partial x_1} |_{x_1 = \hat{x}_1} = 0$, and it should be a local minimum. Similarly, one can show that there exists at most one $\hat{x}_2 > \alpha_2$ for which $\frac{\partial \Pi_2(x_1, x_2)}{\partial x_2} |_{x_2 = \hat{x}_2} = 0$, and it should be a local minimum.

To conclude our equilibrium existence arguments we will show that there exists d' such that, whenever $d < d'$, the Nash equilibrium of the restricted game, (x_1^*, x_2^*), is a Nash equilibrium of the unrestricted game too.

Step 4. Assume that $d \to 0$, that we are in the unrestricted game and that the candidates play the Nash equilibrium of the restricted game, (x_1^*, x_2^*). Candidate 1 has no incentives to deviate: a) to any $x_1 \in [\alpha_1, -d]$ (because (x_1^*, x_2^*) is a Nash equilibrium of the restricted game);

[5] By non-degenerate we mean that it does not converge to zero when $d \to 0$.

[6] To see why this is true consider that α_1 is arbitrarily negative and observe that $\Pi_1(0, b) < \Pi_1(-b, b)$ for any $b > 0$. Log-concavity of F suggests that, for any $\alpha_1 < 0$, $\Pi_1(x_1, b)$ should be strictly decreasing in an interval $[k, 0]$ for some non-degenerate $k < 0$.

b) to any $x_1 \in (-d, x_1(x_2^*, C_1)]$ because $\frac{\partial \Pi_1(x_1, x_2^*)}{\partial x_1} < 0$ in this set; c) to any $x_1 > x_1(x_2^*, C_1)$ because by definition of $x_1(x_2^*, C_1)$, $\Pi_1(x_1(x_2^*, C_1), x_2^*) > \Pi_1(x_1, x_2^*)$; and d) to any $x_1 < \alpha_1$ because $\lim_{x_1 \to -\infty} \Pi_1(x_1, x_2^*) = v(|x_2^* - \alpha_1|) < \Pi_1(\alpha_1, x_2^*)$ and $\Pi_1(x_1, x_2^*)$ is quasi-convex in $(-\infty, \alpha_1]$ (the first follows from the fact that $v(|x_1 - \alpha_1|) - C_1 > v(|x_2^* - \alpha_1|)$ when $x_1 = \alpha_1$ and $d \to 0$ and the second by the arguments in step 3). Therefore, there exists $d' > 0$ such that whenever $d < d'$, the Nash equilibrium of the restricted game, (x_1^*, x_2^*) is an equilibrium of the unrestricted game too. QED

Our setting is similar to the electoral competition model with mixed and asymmetric motives in which the candidates care both about office and about policy but each candidate may put different weight to each of these objectives. The difference is that in our setting, the value of holding office is negative instead of positive. Saporiti (2008) analyzes the case with positive value of holding office and shows that the problem is guaranteed to admit pure strategy equilibria *only in the symmetric case*. That is, only when both candidates value office and policy the same. They may value office more than the policy or vice versa but they both agree on the relative importance of these two goals.[7]

Instead, in the setting with negative value of holding office, we can establish existence of a pure strategy equilibrium much more generally – even for the case in which each candidate values office and policy differently. Indeed, conditional on the costs C_1 and C_2 not being excessively large, the result proved above does precisely that. To understand the intuition for this existence result, consider the candidates' payoff functions. Assume that candidate 2 announces policy x_2 close to his ideal point. When the value of holding office is positive, the payoff of candidate 1 when he approaches x_2 from the left (i.e., when $x_1 \to x_2^-$) is not always decreasing. By approaching x_2, candidate 1 increases the probability of being elected (which increases his payoff given the positive value attached to office) and decreases his expected utility from the implemented policy. Therefore, neither quasi-concavity of the payoff function of candidate 1 in the crucial segment $[\alpha_1, x_2]$ is guaranteed, nor a best-response in that segment is bound to exist. On the contrary, when the value of holding office is negative – like in our case – the payoff of candidate 1 is unambiguously decreasing when $x_1 \to x_2^-$. By approaching x_2, candidate 1 increases the probability of being elected (which is payoff reducing given the negative value attached to office) and decreases his expected utility from the implemented policy. This is very important since it allows us to establish quasi-concavity of the payoff function of candidate 1 in the crucial segment $[\alpha_1, x_2]$ and show that the unique

[7] In the asymmetric case, Drouvelis et al. (2014) provide existence results when the media voter's bliss point is uniformly distributed in a segment of the policy space.

best response of candidate 1 belongs to that set. It means that we are able to provide much more general equilibrium existence results in the case of negative value of holding office.[8]

We want to emphasize that in Proposition 1, we prove the equilibrium existence in the case of sufficiently small costs of policy implementation. In the case of high costs, the candidates value office much more than policy and so – given the negative value of office – prefer to lose the election. Then given any announcement of one candidate, his competitor's best response is to announce a more extreme policy in order to lose the election. But then the candidate himself wants to announce an even more extreme policy in order to lose the elections and so to save the high cost of implementing a policy which is not that valuable (relative to the costs it implies). Given those dynamics, the existence of a pure strategy equilibrium in the case of high costs is not guaranteed.

5. Equilibrium Characterization for Euclidean Preferences

In this section, we characterize an equilibrium and provide comparative statics in the case of Euclidean preferences, $v(|x-\chi|) = -|x-\chi|$ where χ is the ideal point and x is the implemented policy. Equilibrium characterization is presented in the following proposition.

Proposition 2. *An interior equilibrium is such that*

$$(x_1^*, x_2^*) = \left(-\tfrac{C_1}{2} + s - \tfrac{F(s)}{F'(s)}, \tfrac{C_2}{2} + s + \tfrac{1-F(s)}{F'(s)}\right), \tag{5.1}$$

where s is uniquely defined by

$$4F(s) + (C_1 - C_2)F'(s) = 2. \tag{5.2}$$

Proof To characterize an interior equilibrium (x_1^*, x_2^*) we notice that it must be such that: a) $\frac{\partial \Pi_1(x_1,x_2^*)}{\partial x_1} |_{x_1=x_1^*} = 0$ and $\frac{\partial \Pi_2(x_1^*,x_2)}{\partial x_2} |_{x_2=x_2^*} = 0$; and b) $\alpha_1 < x_1^* < x_2^* < \alpha_2$. We observe that

$$\frac{\partial \Pi_1(x_1,x_2^*)}{\partial x_1} |_{x_1=x_1^*} = -F\left(\frac{x_1^*+x_2^*}{2}\right) + \frac{1}{2}(-C_1 - x_1^* + x_2^*)F'\left(\frac{x_1^*+x_2^*}{2}\right) = 0$$

[8] As far as mixed equilibria are concerned, one can adjust arguments of Saporiti (2008), since they do not depend on whether the value of holding office is positive or negative, and show that a mixed strategy equilibrium is guaranteed to exist for any parametrization of our model, considering though a bounded strategy space. Indeed, when a player's strategy space is unbounded a game need not admit an equilibrium neither in pure nor in mixed strategies even when the discontinuities in the payoff functions are "well behaved" (in the sense that the conditions of known equilibrium existence theorems are satisfied; see, for example, Dasgupta and Maskin 1986, Reny 1999, etc.).

and

$$\frac{\partial \Pi_2(x_1^*, x_2)}{\partial x_2} \mid_{x_2 = x_2^*} = 1 - F\left(\frac{x_1^* + x_2^*}{2}\right) + \frac{1}{2}(C_2 + x_1^* - x_2^*) F'\left(\frac{x_1^* + x_2^*}{2}\right) = 0$$

if and only if

$$4F\left(\frac{x_1^* + x_2^*}{2}\right) + (C_1 - C_2) F'\left(\frac{x_1^* + x_2^*}{2}\right) = 2.$$

Log-concavity of $F(\cdot)$ suggests that there exists a unique value of $\frac{x_1^* + x_2^*}{2}$ for which this equality holds. We denote this unique value by s. Substituting s in the first-order conditions yields (5.1). QED

An interior equilibrium exists when

$$-\frac{C_1}{2} + s - \frac{F(s)}{F'(s)} > \alpha_1 \text{ and } \frac{C_2}{2} + s + \frac{1 - F(s)}{F'(s)} < \alpha_2.$$

That is, when the candidates' preferred policies are substantially apart. Note that this is equivalent to the costs C_1 and C_2 being sufficiently small. According to Proposition 2, there is no policy convergence in the equilibrium. Intuitively, neither candidate wants to carry the cost of implementing a policy that can be implemented by the other candidate. The equilibrium strategies (x_1^*, x_2^*) and s depend on policy implementation costs C_1 and C_2. We present the comparative statics with respect to C_1 and C_2 in the following proposition.

Proposition 3. *The equilibrium probability of a candidate being elected is decreasing in his cost of policy implementation and is increasing in his competitor's cost:*

$$\frac{\partial p_i(x_1^*, x_2^*)}{\partial C_i} < 0, \qquad \frac{\partial p_i(x_1^*, x_2^*)}{\partial C_{-i}} > 0.$$

If a candidate's cost of policy implementation is higher than that of his competitor, then an increase in his own cost makes the equilibrium policy announcements of both candidates more extreme, that is, for $C_i > C_{-i}$

$$\frac{\partial x_1^*}{\partial C_i} < 0, \quad \frac{\partial x_2^*}{\partial C_i} > 0;$$

however, an increase in the competitor's cost makes the candidate announce a more moderate policy while the competitor himself announce a more extreme policy for sufficiently small (and hence similar) costs, that is, for $C_i > C_{-i}$

$$\begin{array}{ll} \frac{\partial x_i^*}{\partial C_{-i}} > 0, \quad \frac{\partial x_{-i}^*}{\partial C_{-i}} > 0 & \text{if } i = 1 \\ \frac{\partial x_i^*}{\partial C_{-i}} < 0, \quad \frac{\partial x_{-i}^*}{\partial C_{-i}} < 0 & \text{if } i = 2 \end{array}.$$

If the candidates' costs of policy implementation are equal, then in equilibrium each candidate's policy announcement depends only on his own cost and not on that of his competitor.

The higher the cost, the more extreme policy is announced in equilibrium. That is, for $C_1 = C_2$

$$\begin{aligned} \frac{\partial x_1^*}{\partial C_1} &< 0, \ \frac{\partial x_2^*}{\partial C_2} > 0, \\ \frac{\partial x_1^*}{\partial C_2} &= 0, \ \frac{\partial x_2^*}{\partial C_1} = 0. \end{aligned}$$

Proof Note that (5.2) along with the symmetry of $F(\cdot)$ around zero suggest that: a) if $C_1 > C_2$ then $s < 0$, b) if $C_1 < C_2$ then $s > 0$ and c) if $C_1 = C_2$ then $s = 0$. Moreover,

$$\frac{\partial s}{\partial C_1} = -\frac{F'(s)}{4F'(s)+(C_1-C_2)F''(s)},$$

which is always negative given the above observations a)-c) and the assumptions about $F(\cdot)$. Similarly,

$$\frac{\partial s}{\partial C_2} = \frac{F'(s)}{4F'(s)+(C_1-C_2)F''(s)} > 0.$$

Since $F'(\cdot) > 0$, the equilibrium probability of candidate i being elected $p_i(x_1^*, x_2^*)$ decreases with his cost of policy implementation C_i and increases with his competitor's cost C_{-i}.

Furthermore,

$$\frac{\partial x_1^*}{\partial C_1} = -\frac{1}{2} + \frac{\partial s}{\partial C_1}\frac{F(s)F''(s)}{(F'(s))^2}, \qquad \frac{\partial x_1^*}{\partial C_2} = \frac{\partial s}{\partial C_2}\frac{F(s)F''(s)}{(F'(s))^2},$$

$$\frac{\partial x_2^*}{\partial C_1} = -\frac{\partial s}{\partial C_1}\frac{(1-F(s))F''(s)}{(F'(s))^2}, \qquad \frac{\partial x_2^*}{\partial C_2} = \frac{1}{2} - \frac{\partial s}{\partial C_2}\frac{(1-F(s))F''(s)}{(F'(s))^2}.$$

Given the above observations a)-c) and the assumptions about $F(\cdot)$, the following holds.

1. If $C_1 > C_2$ then $\frac{\partial x_1^*}{\partial C_1} < 0$, $\frac{\partial x_1^*}{\partial C_2} > 0$, $\frac{\partial x_2^*}{\partial C_1} > 0$. Moreover, for sufficiently small (and hence similar) costs C_1 and C_2, $s \to 0$ and so $\frac{\partial x_2^*}{\partial C_2} > 0$.
2. If $C_1 < C_2$ then $\frac{\partial x_1^*}{\partial C_2} < 0$, $\frac{\partial x_2^*}{\partial C_1} < 0$, and $\frac{\partial x_2^*}{\partial C_2} > 0$. For sufficiently small (and hence similar) costs C_1 and C_2, $s \to 0$ and so $\frac{\partial x_1^*}{\partial C_1} < 0$.
3. If $C_1 = C_2$ then $\frac{\partial x_1^*}{\partial C_1} < 0$, $\frac{\partial x_1^*}{\partial C_2} = 0$, $\frac{\partial x_2^*}{\partial C_1} = 0$, and $\frac{\partial x_2^*}{\partial C_2} > 0$. QED

Intuitively, a higher cost of policy implementation implies a higher weight of office relative to policy in the candidate's payoff function and so – given the negative value of office – increases his incentives to lose the election. Then the candidate's chances of being elected decrease as his cost of policy implementation raises. Clearly, his competitor's chances increase in this case. Note however that the effect of a cost change on equilibrium announcements depends on the cost's relative magnitude, i.e., on whether this cost is higher or lower than

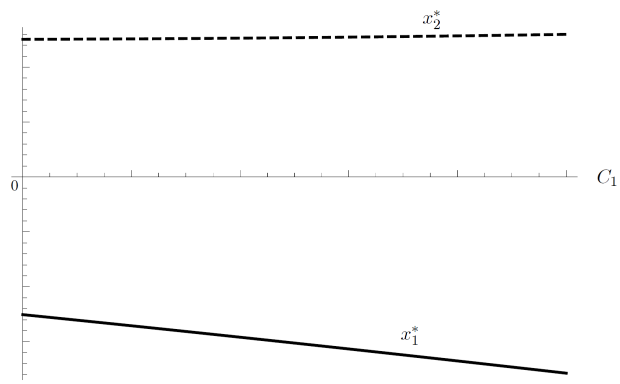

Figure 5.1: Equilibrium policy announcements x_1^* and x_2^* as functions of C_1.

that of the competitor. In particular, an increase in a higher cost makes the equilibrium announcements of both candidates more extreme. An increase in a lower cost makes the corresponding candidate announce a more extreme policy for sufficiently small costs while makes the competitor's policy more moderate. In the case of equal costs of implementation, the equilibrium policy announcement of each candidate depends only on his own cost. A higher cost makes the candidate choose a more extreme policy in this case.

Example Suppose that $F(\cdot)$ has a relatively flat density around zero which takes value one. Formally, $F(\rho) = \rho + \frac{1}{2}$ for ρ in a neighborhood of zero. Then (5.2) yields $s = \frac{C_2 - C_1}{4}$. Therefore, for sufficiently small costs C_1 and C_2, the pair

$$(x_1^*, x_2^*) = \left(-\tfrac{1+C_1}{2}, \tfrac{1+C_2}{2}\right) \tag{5.3}$$

is the interior equilibrium of the game. It follows that

$$\frac{\partial x_1^*}{\partial C_1} < 0, \quad \frac{\partial x_1^*}{\partial C_2} = 0, \quad \frac{\partial x_2^*}{\partial C_1} = 0, \quad \frac{\partial x_2^*}{\partial C_2} > 0.$$

This is a very particular case in which the equilibrium strategy of each candidate depends only on his cost of policy implementation and not on that of the other candidate. Figure 5.1 and Figure 5.2 depict equilibrium announcements and s, respectively, as functions of candidate 1's cost C_1 in this case.

5.1. Median Voter's Welfare

We consider next the expected payoff of the median voter, Ev_{MV}, in the interior equilibrium characterized in Proposition 2. Given that the median voter's preferred policy ρ is distributed

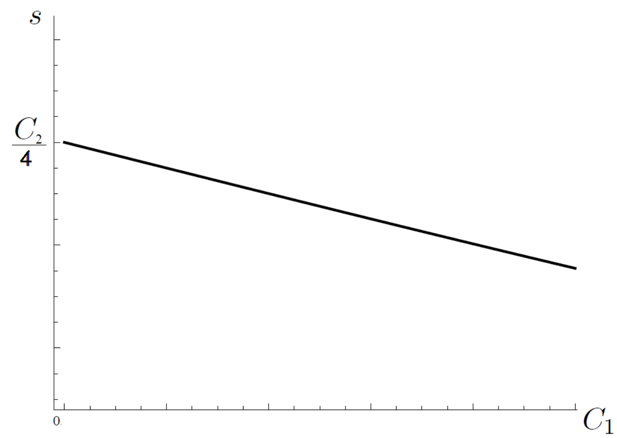

Figure 5.2: s as a function of C_1.

according to $F(\cdot)$, it is equal to

$$Ev_{MV} \equiv \int_{-\infty}^{s} \left(-\left|x_1^* - \rho\right|\right) F'(\rho)\, d\rho + \int_{s}^{+\infty} \left(-\left|x_2^* - \rho\right|\right) F'(\rho)\, d\rho =$$

$$\int_{-\infty}^{x_1^*} \left(\rho - x_1^*\right) F'(\rho)\, d\rho + \int_{x_1^*}^{s} \left(x_1^* - \rho\right) F'(\rho)\, d\rho + \int_{s}^{x_2^*} \left(\rho - x_2^*\right) F'(\rho)\, d\rho + \int_{x_2^*}^{+\infty} \left(x_2^* - \rho\right) F'(\rho)\, d\rho.$$

How does the median voter's welfare depend on the costs of policy implementation C_1 and C_2? Differentiating Ev_{MV} with respect to C_i yields

$$\frac{\partial Ev_{MV}}{\partial C_i} = \int_{-\infty}^{x_1^*} \left(-\frac{\partial x_1^*}{\partial C_i}\right) F'(\rho)\, d\rho + \int_{x_1^*}^{s} \frac{\partial x_1^*}{\partial C_i} F'(\rho)\, d\rho + \int_{s}^{x_2^*} \left(-\frac{\partial x_2^*}{\partial C_i}\right) F'(\rho)\, d\rho + \int_{x_2^*}^{+\infty} \frac{\partial x_2^*}{\partial C_i} F'(\rho)\, d\rho =$$
$$\frac{\partial x_1^*}{\partial C_i} \cdot \left(F(s) - 2F(x_1^*)\right) + \frac{\partial x_2^*}{\partial C_i} \cdot \left(1 + F(s) - 2F(x_2^*)\right).$$

Note that depending on the relationship between the electoral uncertainty (defined by $F(\cdot)$) and the magnitude of costs C_1 and C_2, the sign of $\frac{\partial Ev_{MV}}{\partial C_i}$ may be positive as well as negative.

Example Consider the above example with a relatively flat density around zero ($F(\rho) = \rho + \frac{1}{2}$ for ρ in a neighborhood of zero). Substituting (5.3) into the median voter's expected payoff yields for sufficiently small costs C_1 and C_2

$$\int_{-\frac{1}{2}}^{\frac{C_2 - C_1}{4}} \left(-\frac{1+C_1}{2} - \rho\right) d\rho + \int_{\frac{C_2 - C_1}{4}}^{\frac{1}{2}} \left(\rho - \frac{1+C_2}{2}\right) d\rho = \frac{1}{16}C_1^2 - \frac{1}{8}C_1C_2 - \frac{1}{4}C_1 + \frac{1}{16}C_2^2 - \frac{1}{4}C_2 - \frac{1}{4}.$$

Differentiating it with respect to the implementation costs yields

$$\frac{\partial Ev_{MV}}{\partial C_1} = \frac{1}{8}\left(C_1 - C_2 - 2\right), \quad \frac{\partial Ev_{MV}}{\partial C_2} = \frac{1}{8}\left(C_2 - C_1 - 2\right),$$

which are negative for sufficiently small costs C_1 and C_2. Therefore, in this example, the median voter's expected payoff is decreasing in the implementation costs C_1 and C_2.

Notice next that in the case of equal costs $C_1 = C_2 \equiv C$, the median voter's welfare is decreasing in C for any log-concave symmetric $F(\cdot)$. The following proposition formalizes this finding.

Proposition 4. *In the case of equal costs of policy implementation, the median voter's welfare is decreasing in costs.*

Proof In the case of $C_1 = C_2 \equiv C$, differentiating Ev_{MV} with respect to C yields

$$\frac{\partial Ev_{MV}}{\partial C} = -\frac{1}{2}\left(1 - 4F\left(-\frac{C}{2} - \frac{1}{2F'(0)}\right)\right).$$

Notice that it is negative if

$$C > -2F^{-1}\left(\frac{1}{4}\right) - \frac{1}{F'(0)}. \tag{5.4}$$

We prove next that the right-hand side of (5.4) is negative and so (5.4) holds for every $C > 0$ (which implies that $\frac{\partial Ev_{MV}}{\partial C} < 0$). To prove that the right-hand side of (5.4) is negative, we notice that

$$\frac{\frac{1}{2} - F\left(-\frac{1}{2F'(0)}\right)}{0 - \left(-\frac{1}{2F'(0)}\right)} > F'\left(-\frac{1}{2F'(0)}\right)$$

due to unimodality and symmetry of $F(\cdot)$. It amounts to

$$\frac{1}{2} - F\left(-\frac{1}{2F'(0)}\right) > \frac{F'\left(-\frac{1}{2F'(0)}\right)}{2F'(0)}. \tag{5.5}$$

Furthermore, due to log-concavity of $F(\cdot)$,

$$\frac{F'\left(-\frac{1}{2F'(0)}\right)}{F\left(-\frac{1}{2F'(0)}\right)} \geq \frac{F'(0)}{F(0)} \quad \Rightarrow \quad \frac{F'\left(-\frac{1}{2F'(0)}\right)}{2F'(0)} \geq F\left(-\frac{1}{2F'(0)}\right). \tag{5.6}$$

Combining (5.5) and (5.6) yields

$$\frac{1}{2} - F\left(-\frac{1}{2F'(0)}\right) > F\left(-\frac{1}{2F'(0)}\right) \quad \Rightarrow \quad -2F^{-1}\left(\frac{1}{4}\right) - \frac{1}{F'(0)} < 0.$$

Thus, the right-hand side of (5.4) is negative and so $\frac{\partial Ev_{MV}}{\partial C} < 0$. QED

Intuitively, the higher the cost of policy implementation, the more extreme policy the corresponding candidate announces. This in turn increases the median voter's expected loss from the implemented policy and so decreases her expected payoff in equilibrium.

5.2. Convergence to the Certainty Case

How does the interior equilibrium look like when the distribution of the median voter's bliss point, $F(\cdot)$, converges to the certainty case? The following proposition presents the results for this case.

Proposition 5. *If* $(F_k(\cdot))_{k=1}^{\infty}$ *is a sequence of admissible distributions such that* $F_k'(0) \to +\infty$, $F_k(\rho) \to F_k'(\rho) \to 0$ *for* $\rho < 0$ *and* $\frac{F_k(\rho)}{F_k'(\rho)} \to 0$ *for* $\rho \leq 0$[9] *then*

$$(x_1^*, x_2^*) \to \left(-\tfrac{\max[C_1,C_2]}{2}, \tfrac{\max[C_1,C_2]}{2}\right).$$

Proof If $C_1 = C_2 = C$ then $s_k = 0$ for every k because

$$4F_k(s_k) + (C_1 - C_2)F_k'(s_k) = 2$$

yields $F_k(s_k) = \frac{1}{2}$. Hence, $(x_1^*, x_2^*) \to \left(-\frac{C}{2}, \frac{C}{2}\right)$.

If $C_1 \neq C_2$ then assume that there exists $\varepsilon > 0$ such that for every k there exists $\hat{k} > k$ with $s_{\hat{k}} \notin [-\varepsilon, \varepsilon]$. Then there exists a subsequence $(\tilde{F}_k(\cdot))_{k=1}^{\infty}$ such that: a) $\tilde{F}_k'(0) \to +\infty$ and $\tilde{F}_k(\rho) \to \tilde{F}_k'(\rho) \to 0$ for every $\rho < 0$; and b)

$$4\tilde{F}_k(s_k) + (C_1 - C_2)F_k'(s_k) = 2$$

with $s_k \notin [-\varepsilon, \varepsilon]$ for every k. But if a) holds then

$$4\tilde{F}_k(s_k) + (C_1 - C_2)\tilde{F}'(s_k) \to 0 \neq 2$$

and so b) doesn't hold. Therefore, there exists no $\varepsilon > 0$ such that for every k there exists $\hat{k} > k$ with $s_{\hat{k}} \notin [-\varepsilon, \varepsilon]$. Hence $s_k \to 0$. Notice next that for $C_1 > C_2$

$$4F_k(s_k) + (C_1 - C_2)F_k'(s_k) = 2$$

yields

$$4\frac{F_k(s_k)}{F_k'(s_k)} + (C_1 - C_2) = \frac{2}{F_k'(s_k)}.$$

This suggests that $F_k'(s_k) \to \frac{2}{C_1 - C_2}$ and hence $(x_1^*, x_2^*) \to \left(-\frac{C_1}{2}, \frac{C_1}{2}\right)$. Similarly, when $C_1 < C_2$ we have that $(x_1^*, x_2^*) \to \left(-\frac{C_2}{2}, \frac{C_2}{2}\right)$. QED

According to Proposition 5, as the distribution of the median voter's bliss point converges to the certainty case, the candidates' announcements are symmetric around 0 in interior

[9] Note that the condition $\frac{F_k(\rho)}{F_k'(\rho)} \to 0$ for $\rho \leq 0$ holds for large families of distributions such that normal and logistic.

equilibrium. Intuitively, suppose that the candidates announce asymmetric policies. Then a candidate with the more moderate policy has incentives to deviate to a more extreme policy closer to his bliss point, which would generate a higher policy payoff for him. These incentives to deviate arise whenever the candidates propose asymmetric policies, which implies that the interior equilibrium does not converge to asymmetric announcements.

We have also analyzed the case of electoral certainty (available upon request) in which the candidates observe the median voter's bliss point. In this case, we consider the median voter as a strategic player with bliss point 0. We show that for sufficiently low costs C_1 and C_2, there exists a continuum of subgame perfect equilibria in which the candidates announce symmetric policies $(-x^*, x^*)$ such that $\frac{\min[C_1,C_2]}{2} \leq x^* \leq \frac{\max[C_1,C_2]}{2}$ and the median voter elects the lower cost candidate.[10] These equilibria are not payoff equivalent. When $x^* = \frac{\max[C_1,C_2]}{2}$, the lower cost candidate receives the highest utility while the higher cost candidate receives the lowest utility. Note that this is the equilibrium to which the electoral uncertainty case converges (as proved in Proposition 5).

6. Conclusion

This paper analyzes a spatial model of electoral competition between two candidates who care both about the policy and office. The novelty is to assume that an elected candidate has to incur a cost in order to gain influence and implement his announced policy. Therefore, in our setting, the value of holding office is negative while the existing literature has assumed positive value of office.

Our main contribution is to prove that a pure strategy Nash equilibrium is guaranteed to exist even for heterogeneous costs of policy implementation as long as they are not very large. This existence result is much more general than that for the case of positive value of holding office in which cost homogeneity is crucial for establishing existence of a pure strategy Nash equilibrium. We also provide equilibrium characterization, comparative statics and discuss the welfare properties of the equilibrium. We show furthermore how the equilibrium looks like when the distribution of the median voter's bliss point converges to the certainty case.

From a more general perspective, our study emphasizes the impacts of policy implementation costs on electoral competition. This emphasis allows us to disclose novel mechanisms and incentives faced by the candidates in the case of costly policy implementation. Therefore, our findings complement the existing literature and suggest that future research may benefit from taking these novel mechanisms and incentive dynamics into account.

[10] Formally, these equilibria exist for $\alpha_1 < -\frac{\max[C_1,C_2]}{2} < \frac{\max[C_1,C_2]}{2} < \alpha_2$.

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